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Numerical Simulation of Three-Dimensional Supersonic Free Shear Layers

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Abstract

THE temporal stability and growth characteristics of three-dimensional supersonic shear layers are numerically investigated. An explicit time-marching scheme that is second-order accurate in time and fourth-order accurate in space is used to study this problem. The shear layer is excited by instability waves computed from a linear stability analysis and random initial disturbances. At low convective Mach numbers, organized vortical structures develop both for the random disturbance and the modal disturbance cases. At supersonic convective Mach numbers, vortical structures develop initially but are not sustained in time. Temporal growth of disturbances is found to be a strong function of the convective Mach number.

Contents

An improved understanding of factors that contribute to supersonic shear-layer growth is necessary for design of active and passive control techniques to enhance the mixing of airstreams and fuel streams, and for the design of efficient, compact SCRAMJET engines. It has been observed[1-3] that, in supersonic shear layers, organized vortical structures exist in a manner similar to subsonic shear layers. However, as the convective Mach number increases, the streamwise shear-layer growth rate is found to drop to about 30% of that of an incompressible flow.[4]

In the past, Tang et al.[5] used a fourth-order MacCormack scheme to study temporal and spatial growth of two-dimensional thin shear layers at very early stages of laminar mixing, and studied the effects of convective Mach number as well as streamwise, spanwise, and cross-stream velocity disturbances on the shear layer growth. It was demonstrated that the growth rate of the shear layer decreased with increasing convective Mach number. In this work, three-dimensional, temporarily growing mixing layers have been studied. The study focuses on the effects of instability waves computed using a linear stability analysis and random initial disturbances on a temporarily evolving shear layer.

The three-dimensional, laminar, unsteady, compressible flow is governed by the Navier-Stokes equations, which may be formally written in a strong conservation form

$$q_t + F_x + G_y + H_z = R_x + S_y + T_z \tag{1}$$

where F, G, and H are inviscid flux terms, and R, S, and T are the viscous stress terms. Equation (1) was solved using an operator splitting approach and a MacCormack-type finite difference scheme:

$$q^{n+2} = L_x L_y L_z L_{xv} L_{yv} L_{zv} L_{zv} L_{yv} L_{xv} L_z L_y L_x q^n \tag{2}$$

The L_x, L_y, and L_z operators involve solutions of the one-dimensional equation such as

$$q_t + F_x = 0 \tag{3}$$

This one-dimensional equation is solved through the following predictor-corrector sequence, recommended by Bayliss and Maestrello.[6]

Predictor step:

$$q_i^* = q_i^n - \frac{Dt}{6\,Dx}[7F_i - 8F_{i-1} + F_{i-2}]^n \tag{4}$$

Corrector step:

$$q_i^{n+1} = \frac{(q_i^* + q_i^n)}{2} + \frac{Dt}{12\,Dx}[7F_i - 8F_{i+1} + F_{i+2}]^* \tag{5}$$

The viscous operators L_{xv}, L_{yv}, and L_{zv} are integrated similarly with the exception that the viscous stress terms are differenced in space with the second-order-accurate central difference scheme. The overall numerical scheme is fourth-order accurate in space and second-order accurate in time as far as the inviscid part is concerned.

Since the temporal development mixing layers are studied, periodic boundary conditions in the streamwise and spanwise directions and slip boundary conditions in the cross-stream direction are applied.

The computational domain is a rectangular channel that extends in stream- and spanwise directions over one wavelength of the longest disturbance wave predicted by linear stability analysis for a given convective Mach number. In the cross-stream direction, it extends from −7.5 to 7.5 times the vorticity thickness. The computational domain is discretized with a 66 × 34 × 121 uniformly spaced grid along the streamwise, spanwise, and cross-stream directions, respectively. The Reynolds number is based on the vorticity thickness and

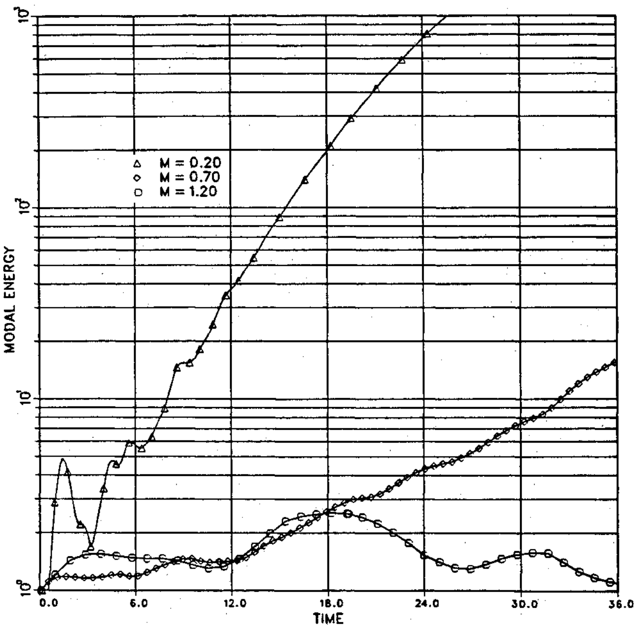

Fig. 1 Modal energy growth of the most unstable modes in shear layers disturbed by instability waves.

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ranges between 3×10^2 and 6×10^2. The mean velocity across the cross-stream direction is given by a hyperbolic tangent profile. The convective Mach number is defined as $M_c = (U_1 - U_2)/(c_1 + c_2)$, where U is the magnitude of the mean velocity, c is the speed of sound, and subscripts 1 and 2 refer to the upper and lower streams, respectively.

The modal kinetic energy content of the flowfield is defined as

$$E_{mn}(t) = \int [uu^* + vv^* + ww^*] \, dz \tag{6}$$

where $\boldsymbol{u}$, $\boldsymbol{v}$, and $\boldsymbol{w}$ are the two-dimensional Fourier transforms of the velocity field on the plane that spans in the streamwise and spanwise directions. The integration is in the cross-stream direction. The superscript * denotes the complex conjugate.

Instability Waves Superposed on Mean Flow

We have first superposed three-dimensional waves onto the mean flow and monitored temporal evolution of the flowfield and the modal energy growth. These disturbance waves are the most unstable waves predicted by the linear stability analysis[3] at a given convective Mach number and are given for the velocity components, density, and temperature in the following form:

$$d(x,y,z) = AD(z) \exp i(\alpha x + \beta y) \tag{7}$$

where $D(z)$ is the eigenfunction, α and β are the wave numbers, and A is the magnitude, which is set to $0.015M_c$.

We have studied the flowfields for convective Mach numbers of 0.2, 0.7, and 1.2. The corresponding wavelengths of the most unstable waves, which are predicted by the linear stability analysis code, are $\alpha = 0.41$ and $\beta = 0$ for $M_c = 0.2$, $\alpha = 0.3$ and $\beta = 0.3$ for $M_c = 7$, and $\alpha = 0.14$ and $\beta = 0.07$ for $M_c = 1.2$. The temporal growth of modal kinetic energy associated with the most unstable modes is shown in Fig. 1.

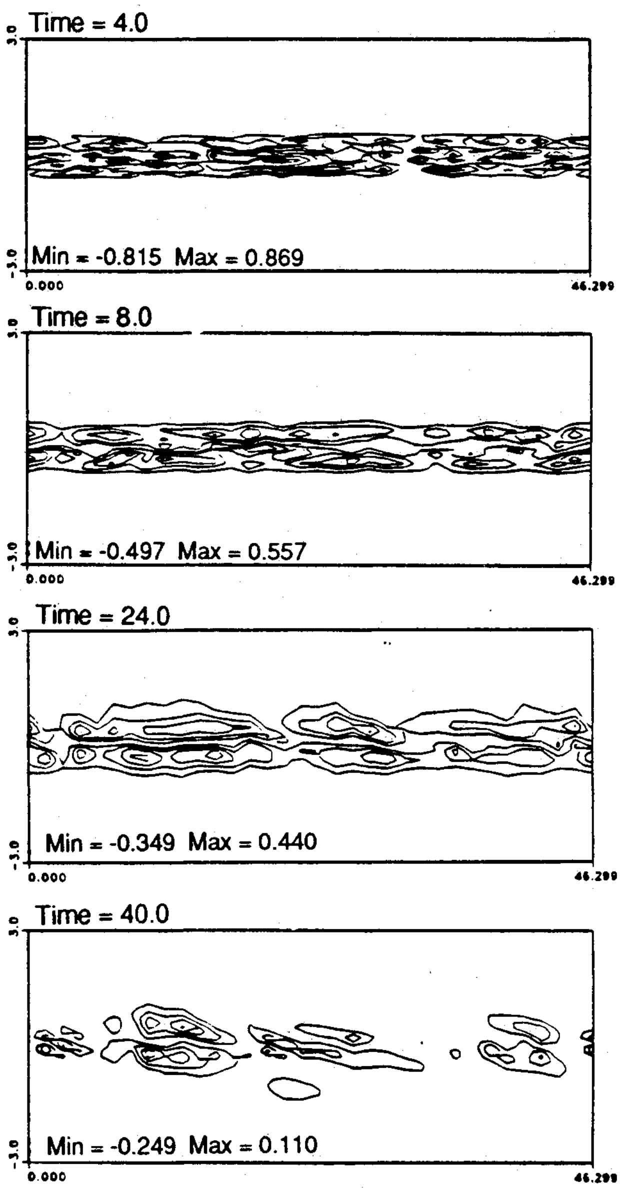

Fig. 2 Spanwise vorticity contours at midspan in a randomly disturbed shear layer, $M_c = 1.2$.

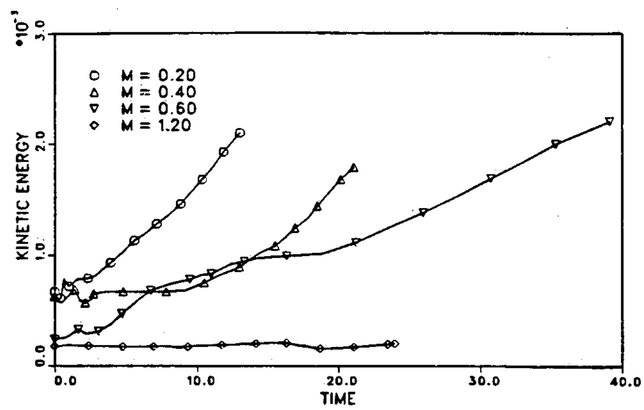

Fig. 3 Average perturbation kinetic energy growth in randomly disturbed shear layers.

Random Disturbances Superposed on Mean Flow

Next, we superposed a random initial disturbance field onto the mean flowfield. The random disturbance field was generated using a random number generator, and its magnitude at any point in the flowfield was restricted to be less than $0.03M_c$. These disturbances were confined to regions of significant vorticity in the shear layer, where $|u(y)| \leq 0.25M_c$. Computations were performed for convective Mach numbers of 0.2, 0.4, 0.6, and 1.2.

In all flow cases, random organized vortical structures were observed in the perturbation velocity field. However, at a higher convective Mach number of 1.2, organized structures tended to die out in time (Fig. 2). The temporal growth of average perturbation kinetic energy for the cases studied is given in Fig. 3.

The following is concluded:

1) The temporal growth rate in perturbation kinetic energy decreases with increasing convective Mach numbers for both modal and random disturbances.

2) At supersonic convective Mach numbers ($M_c = 1.2$), the growth of three-dimensional structures were also found to be unsustainable in both the random and modal disturbance cases.

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